Mimetic reformulation of finite element methods and what it can do for you

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A recent study [1] reveals that convergence of finite element methods employing $H(\Omega, \text{div})$ compatible finite element spaces deteriorates on non-affine quadrilateral grids. This deterioration is particularly troublesome for the lowest-order Raviart-Thomas element, because it implies loss of convergence in some norms for finite element solutions of mixed and least-squares methods. In this talk we show that reformulation of finite element methods in terms of the so-called natural mimetic divergence operator [2] restores the order of convergence.

Reformulations of mixed Galerkin and least-squares methods for the Darcy equation illustrate our approach. We prove that reformulated methods converge optimally with respect to a norm involving the mimetic divergence operator. Furthermore, we prove that standard and reformulated versions of the mixed Galerkin method actually *coincide*, but that the two versions of the least-squares method are genuinely different. The surprising conclusion is that the degradation of convergence in the mixed method on non-affine quadrilateral grids is superficial, and that the lowest order Raviart-Thomas elements are safe to use in this method. However, the breakdown in the least-squares method is real, and there one should use our proposed reformulation.

REFERENCES

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